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ELECTROHYDRODYNAMIC DROPLET GENERATORS

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Summary

In an electrohydrodynamic (EHD) droplet generator, an electric field acts directly on the surface of an electrically conducting jet after it has been formed and ejected from the nozzle. If the electric field has the proper frequency, it will induce compression and expansion of the jet, which will eventually break up into droplets some distance downstream. In contrast to acoustic droplet generators, the initial disturbance is physically separated from the nozzle, so that the exciter and nozzle can be individually designed to perform their separate tasks efficiently. This paper presents a theory of EHD exciters which predicts the breakup length of the jet in terms of the geometry and physical properties of the printer. Experiments confirm this theory, and show that practical exciters can be designed from first principles with the expectation that the breakup length will be close to that predicted, and insensitive to changes in operating frequency.

1. Introduction

Liquid jet breakup into controlled drops plays a crucial role in many commercial and research devices. The ink jet printer, for example, requires the formation of an ink drop at a precisely determined point at the correct time, a requirement which is usually met by jet breakup [1]. Research studies in meteorological processes often include the collision and coalescence of water droplets formed by jet breakup [2]. This process even plays a role in the sorting of individual biological cells, which are trapped within the drops, and deflected to the desired destination [3].

Most droplet generators depend on electromechanical (acoustic) vibrations to introduce the initial perturbation which is then amplified to form droplets. Acoustic excitation involves the transmission of sound waves through the nozzle assembly, which is usually many wavelengths long at the excitation frequency. These waves are reflected back and forth, interfering with each other to produce an extremely complicated pattern. Because of this standing wave pattern, an acoustically excited jet exhibits sharp peaks and nulls in its response as operating parameters such as frequency are changed by less than one percent.

Electric forces acting on the surface of a formed jet can also stimulate

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droplet production. This method was first reported by Sweet [4], but there has never been a theoretical analysis of EHD excitation of the droplet mode, although EHD excitation of the $m = 1$ mode has been studied in some detail [5]. From this earlier work, it seems clear that any mode may be excited electrostatically. The present work, however, concentrates on the $m = 0$, or droplet mode, which plays a key role in droplet generators [6].

2. The linear, long wave model

The long wave cylindrical jet model described by Lee [7] offers a useful starting point for the exciter theory. The geometry of the jet is defined by Fig. 1, which shows a circular liquid jet entering an electrode. The jet has a radius R, and moves with a velocity U_0. This particular electrode is shown as a cylinder, but it can have any shape, since the theory assumes only that the electric field at the surface of the jet can be calculated. With the neglect of viscosity and body forces, the momentum equation for motion in the axial direction can be written as

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} = -\frac{1}{\rho} \frac{\partial}{\partial x} (p_s + p_e) , \tag{1}$$

while Conservation of Mass for a round jet is given by

$$\frac{\partial R}{\partial t} + U \frac{\partial R}{\partial x} + \frac{R}{2} \frac{\partial U}{\partial x} = 0 . \tag{2}$$

The surface tension pressure in the long wave model is

$$p_s = T \frac{\dfrac{1}{R} - \dfrac{\partial^2 R/\partial x^2}{1 + (\partial R/\partial x)^2}}{\sqrt{1 + (\partial R/\partial x)^2}} \tag{3}$$

The electric pressure term appears in the jet equation for the first time here.

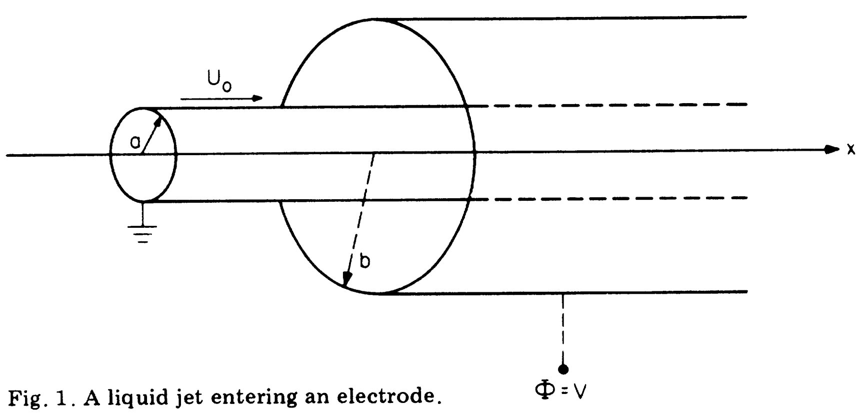

Fig. 1. A liquid jet entering an electrode.

The electrical pressure inside a conducting jet is given in terms of the electric field at the surface as [8]

$$p_e = -\tfrac{1}{2}\,\epsilon\, E^2 \; . \tag{4}$$

These equations, although simpler than the full Navier—Stokes equations, are still non-linear. Inside the exciter, however, the disturbance is very small, and linearization of the equation offers an opportunity to simplify the model still further [9]. Introducing

$$\begin{aligned} U &= U_0 + u \qquad u \ll U_0 \\ R &= a + \delta \qquad \delta \ll a \end{aligned} \tag{5}$$

into the long wave equations, and neglecting second-order terms yields the linearized equations for the surface displacement as

$$\left(\frac{\partial}{\partial t} + U_0 \frac{\partial}{\partial x}\right)^2 \delta = \frac{-T}{2\rho a}\left[\frac{\partial^2 \delta}{\partial x^2} + a^2 \frac{\partial^4 \delta}{\partial x^4}\right] + \frac{a}{2\rho}\frac{\partial^2 p_e}{\partial x^2}\bigg|_{r=a} \tag{6}$$

The fate of the electric pressure term in the linearization process needs some explanation, since some of the pressure terms have been dropped. There are two sources of variation in electric pressure at the jet surface. The most important source is the exciter electrode, which is driven with a time-varying voltage, and extends only over some sections of the jet. These variations generate the disturbance which will grow to form drops; as such, they cannot be neglected in a theory of excitation. They are, therefore, formally included as the last term in eqn. (6). The exact nature of this term depends on the expression for electric pressure, which will be known only after the geometry of the exciter electrode has been selected.

A second source of perturbations is the influence of changes in the jet on the applied electric field, which can affect the growth rate of the droplets [8] and can even excite parametric oscillations [10] on the jet. In exciter practice these effects are relatively unimportant, because the electric pressure is relatively weak compared to surface tension, i.e.,

$$\frac{\epsilon a E^2}{T} \ll 1 \; . \tag{7}$$

Thus these electric pressure terms arising from the jet perturbations have been discarded in the linearized equation. This neglect has the added benefit that all of the coefficients of δ are constant, a result which greatly simplifies the solution.

One convenient method of solution involves sinusoidal steady state in time and Laplace transforms in space. This is appropriate for most exciters, which are driven by sinusoidal voltages, and surround just a finite length of the jet. With these assumptions, the surface displacement will have the form

$$\delta = \mathrm{Re}[\mathrm{e}^{\mathrm{i}\omega t} \textstyle\int \hat{\delta}(s)\mathrm{e}^{sx}\,\mathrm{d}x] \; . \tag{8}$$

Substitution into eqn. 6 and solving gives the response as

$$\hat{\delta}(s) = \frac{\left(\dfrac{a}{2\rho}\right) s^2 \hat{p}_e \Big|_{r=a}}{\dfrac{T}{2\rho a}(s^2 + a^2 s^4) + (i\omega + U_0 s)^2}. \tag{9}$$

Inside the EHD exciter, some of the terms in this equation may often be neglected. Typically, the jet is moving much faster than the capillary velocity, so that

$$\alpha^2 = \frac{T}{2\rho a U_0^2} \ll 1 \tag{10}$$

and terms containing T may be neglected. In the absence of surface tension, the response is limited only by the inertia of the jet, and the equation reduces to the inertial limit:

$$\hat{\delta}(s) = \frac{as^2/(2\rho U_0^2)}{[s + (i\omega/U_0)]^2} \hat{p}_e \Big|_{r=a} \tag{11}$$

Depending on the desired application of the theory, any of the progressively simpler models described above can be used to predict jet behavior. In the following sections, some of these models will be used to describe the behavior of the jet downstream of an exciter, the step response inside the exciter, and the disturbance introduced at the exit of a finite cylindrical exciter.

3. Jet behavior in regions of uniform electric pressure

The drive term in the general linearized equation is proportional to the second spatial derivative of the pressure. In many practical situations, however, such as far downstream of the exciter, or well within a uniform exciter electrode, the electric field is independent of x, so that the solution consists only of the natural response, obtained from eqn. (9) in the form

$$\hat{\delta} = \mathrm{Re}[\delta\, e^{i(\omega t - kx)}]. \tag{12}$$

The quantities ω and k are related by the dispersion relation

$$\alpha^2 (ka)^4 - (1+\alpha^2)(ka)^2 + 2\left(\frac{\omega a}{U_0}\right)(ka) - \left(\frac{\omega a}{U_0}\right)^2 = 0\,. \tag{13}$$

In a typical exciter, the jet velocity is much greater than the capillary velocity ($\alpha^2 \ll 1$), so the dispersion relation has the approximate solution

$$ka \simeq \omega a/U_0 + i\mu a \tag{14}$$

where

$$(\mu a)^2 = \alpha^2 \left(\frac{\omega a}{U_0}\right)^2 \left[1 - \left(\frac{\omega a}{U_0}\right)^2\right] \tag{15}$$

is the growth rate equation for the droplet mode.

Then the surface displacement in the uniform field region can be written as

$$\hat{\delta} = (A_1 \cosh \mu x + A_2 \sinh \mu x) e^{-i\omega x / U_0} \ . \tag{16}$$

The convective radial velocity of the jet surface is also of interest; it is given by

$$\delta' = \frac{\partial \delta}{\partial t} + U_0 \frac{\partial \delta}{\partial x} \ . \tag{17}$$

For the displacement of eqn. (16) this velocity becomes

$$\hat{\delta}' = (\mu U_0 A_1 \sinh \mu x + \mu U_0 A_2 \cosh \mu x)\, e^{-i\omega x / U_0} \ . \tag{18}$$

The constants A_1 and A_2 are determined by the conditions at the upstream boundary, where the disturbance in surface displacement and in velocity are denoted by

$$\hat{\delta}(x{=}0) = \delta_0 \tag{19}$$

and

$$\hat{\delta}'(x{=}0) = \delta_0' \ . \tag{20}$$

Using these conditions, the downstream amplitude of the droplet mode is given by

$$\hat{\delta} = \left(\delta_0 \cosh \mu x + \frac{\delta_0'}{\mu U_0} \sinh \mu x\right) e^{-i\omega x / U_0} \tag{21}$$

Thus there are two waves excited on the jet. One is proportional to the initial surface displacement while the other is proportional to the initial surface velocity. Far downstream, where

$$\mu x \gg 1 \tag{22}$$

the magnitude of the displacement becomes

$$|\hat{\delta}| \rightarrow \frac{1}{2}\left(\delta_0 + \frac{\delta_0'}{\mu U_0}\right) \exp(\mu x) \ . \tag{23}$$

4. The step response to a semi-infinite exciter

A basic building block in complicated exciters is the step response to a semi-infinite cylindrical exciter.

$$p_e = \begin{cases} 0 & x < 0 \\ -P_0 & x > 0 \end{cases} . \tag{24}$$

The minus sign reflects the negative pressure (or tension) resulting from the mutual repulsion of electric charges on the jet surface. The Laplace transform of the drive is

$$\hat{p}(s) = -P_0/s . \tag{25}$$

The response transform is obtained by solving eqn. (11) as

$$\hat{\delta}(s) = -\frac{aP_0}{2\rho U_0^2} \frac{s}{(s + \mathrm{i}\omega/U_0)^2} \tag{26}$$

with the inertial assumption discussed above. This corresponds to

$$\hat{\delta}(x) = -\frac{aP_0}{2\rho U_0^2} (1 - \mathrm{i}\omega x/U_0)\, \mathrm{e}^{-\mathrm{i}\omega x/U_0} . \tag{27}$$

The transverse velocity transform is obtained from eqn. (17) as

$$\hat{\delta}'(s) = U_0(s + \mathrm{i}\omega/U_0)\hat{\delta}(s) . \tag{28}$$

Using this definition gives the velocity transform for a spatial step as

$$\hat{\delta}'(s) = -\frac{aP_0}{2\rho U_0} \frac{s}{(s + \mathrm{i}\omega/U_0)} \tag{29}$$

which corresponds to a velocity of

$$\delta'(x) = -\frac{aP_0}{2\rho U_0} \left[\Delta(x) + \frac{\mathrm{i}\omega}{U_0} \exp\left(-\frac{\mathrm{i}\omega x}{U_0}\right)\right] \qquad x \geqslant 0 \tag{30}$$

The impulse (Δ) in the velocity response represents the quick contraction of the jet as it enters the electrode. It has no effect downstream.

In eqn. (23), the total disturbance in the uniform field region downstream of the exciter was proportional to a combination of displacement and velocity at the exciter, namely,

$$|\delta| \simeq \delta_0 + \delta_0'/\mu U_0 . \tag{31}$$

These two quantities often differ greatly in magnitude. For the basic step response, the magnitude of the two terms at some point $x = l$ is given from eqns. (27), (30) as

$$|\delta_0| \simeq \left|1 - \frac{\mathrm{i}\omega l}{U_0}\right| \frac{aP_0}{2\rho U_0^2} \tag{32}$$

and

$$|\delta_0'| \simeq \frac{\omega a P_0}{2\rho U_0^2} \tag{33}$$

If the exciter length is on the order of the droplet wavelength, which is almost always true in practice,

$$\omega l / U_0 \simeq 1 \tag{34}$$

the ratio of the displacement and velocity terms is approximately

$$\frac{|\delta_0|}{|\delta_0' \mu / U_0|} \simeq \frac{\mu U_0}{\omega} \ll 1 \ , \tag{35}$$

since the growth over one wavelength (U_0/ω) is usually very small. Thus the largest contribution to the droplet mode will usually come from the transverse velocity excited on the jet. This result often proves useful in designing EHD exciters, since the velocity output alone gives a very good indication of how efficient the exciter will be, and the displacement need be calculated only when finer distinctions must be made.

5. Example: finite cylindrical exciter

The semi-infinite exciter described above gives much insight into EHD excitation, but does not serve as a worthwhile model for a device. In practice, most exciters have a finite length, and the effect of this length on the response has to be considered. The simplest finite exciter would be a uniform circular cylinder so close to the jet that fringing is negligible. With this arrangement, the electric pressure inside the jet is

$$p_e = -\tfrac{1}{2} \epsilon E^2 = -\frac{\epsilon V^2(t)}{2a^2 \ln^2(b/a)} \ . \tag{36}$$

Because the electric pressure depends on the square of the voltage, there will be several Fourier components, even when a pure sinusoidal voltage is applied. This poses no difficulty in the solution, for every component in a linear system is independent. The value of voltage is to use in the pressure calculation is the peak value of whatever component is under consideration. For example, if $V(t) = V_{dc} + V_{ac} \cos \omega t$, the instantaneous pressure takes the form

$$p_e = -\frac{\epsilon}{2a^2 \ln^2(b/a)} \left[(V_{dc}^2 + V_{ac}^2/2) + 2 V_{dc} V_{ac} \cos \omega t + V_{ac}^2 \cos 2\omega t \right] . \tag{37}$$

The pressure component at the frequency ω may be written as

$$P_0 = -\frac{\epsilon V_{eff}^2}{2a^2 \ln^2(b/a)} \tag{38}$$

where the effective voltage at that frequency is

$$V_{eff} = (2 V_{dc} V_{ac})^{1/2} \ . \tag{39}$$

Other frequency components can be handled similarly.

The velocity output for this exciter can be obtained from the semi-infinite unit step exciter by superposing a positive step at $x = 0$ and a negative step at $x = l$ to obtain

$$|\delta_0'| = \frac{a\omega P_0}{\rho U_0^2} \left| \sin \frac{\omega l}{2U_0} \right| \qquad (40)$$

for the magnitude of the transverse velocity at the exit, which is most effective in exciting the downstream wave. One of the most striking predictions of this result is the futility of increasing the electrode length to obtain stronger excitation. The length appears only as the argument of a sine function, and indiscriminately increasing the length can only cause periodic peaks and nulls in the response. Best results are obtained when the length is selected to be an odd multiple of a half wavelength. For most purposes, the shortest length which satisfies this condition would be selected, so that

$$l = \pi U_0 \omega \ . \qquad (41)$$

Usually, this length will not be very much greater than the jet diameter.

Another important prediction of the theory is the relatively smooth frequency response of the exciter. The frequency occurs in the output as a linear term multiplied by sinusoid. Neither of these terms varies drastically, and the response is especially smooth near the peak of the sinusoid, which is the most likely operating point. Thus the EHD exciter will not suffer from the frequency sensitivity which often plagues acoustic exciters.

Several experiments [11] were performed in order to test these predictions. The nozzle and associated apparatus were mounted on a specially designed frame which allowed adjustment and positioning suitable for the experiments. The fluid used in most of these experiments had a density of 1030 kg/m^3 and a surface tension of 39 mN/m. The jet formed by the nozzle had a radius of 16.5 μm. Its velocity was determined by measuring the wavelength of the disturbance at a known frequency.

The exciter electrodes used in most of these experiments consisted of steel plates of various nominal thicknesses (3—7 mil, or 75—175 μm) through which holes were drilled. The holes ranged in nominal diameter from 3—7 mils (75—175 μm). Microscopic examination of the electrodes revealed that the holes were essentially straight sided, although occasional burrs could be seen.

The simple theory presented above predicts maxima and minima in the frequency response of the exciters. In order to test these predictions, frequency sweeps were performed on a number of exciters. The easiest feature of the response to check is the null which occurs when the frequency corresponds to an entire wavelength of the disturbance inside the jet at any time. Figure 2 depicts the frequency response for an electrode which is 7 mils (178 μm) thick. For this length, the null is expected at 108 kHz. Since this null occurs near the maximum growth rate of the jet, it could be observed

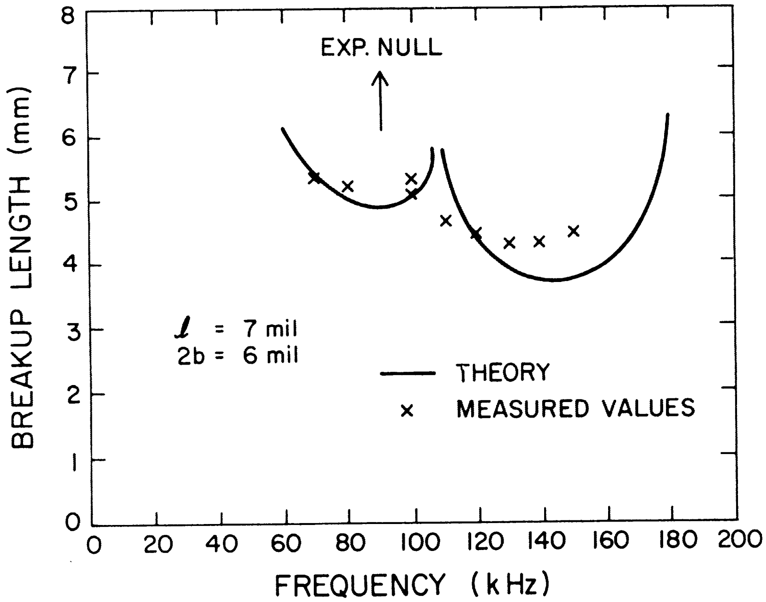

Fig. 2. Frequency response for a 7 mil exciter.

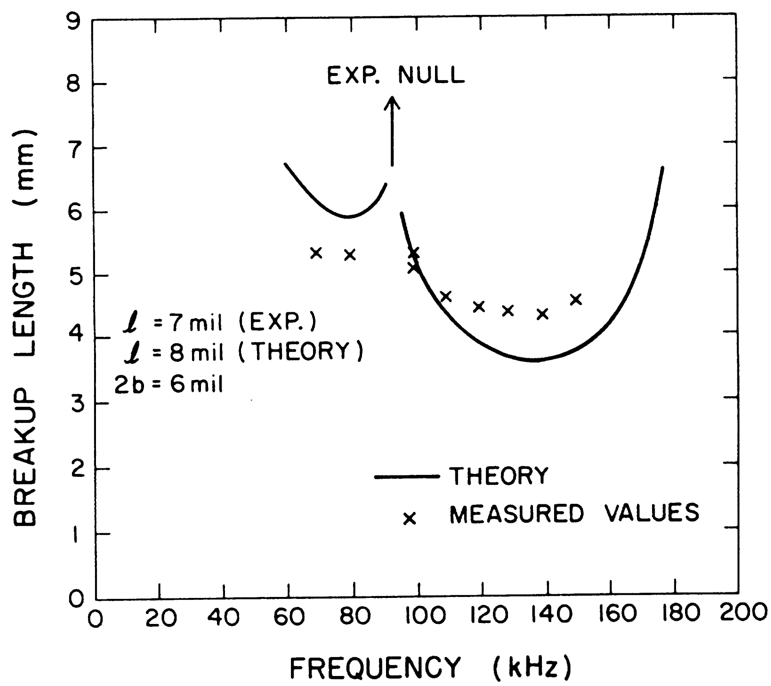

Fig. 3. Same thickness, but 8 mil theory.

experimentally by varying the frequency to produce the longest apparent breakup length. This null frequency (shown in Fig. 2 by an arrow) is lower than expected from theory by approximately 15%. Increasing the effective legth of the exciter from 7 to 8 mils gave better agreement, as shown in Fig. 3. This extension in length might be justified by the presence of burrs at either end of the hole, since such burrs were often observed in microscopic examination of the holes. Another possibility is the effect of fringing in extending the effective length of the electrode beyond the physical limits of the conductor.

This exciter exhibits the predicted null in the frequency response, but does not test the theory at the thickness which is expected to give the shortest breakup length. Since short breakup length is desired in practical printers, the frequency response of a shorter (3 mil or 76 μm) electrode was also measured. The results of this measurement, along with the predictions of the theory, are shown in Fig. 4. Just as in the earlier measurements, the theory predicts both the magnitude and the shape of the frequency response quite well (±0.2 mm or 10%). To appreciate this agreement, it should be kept in

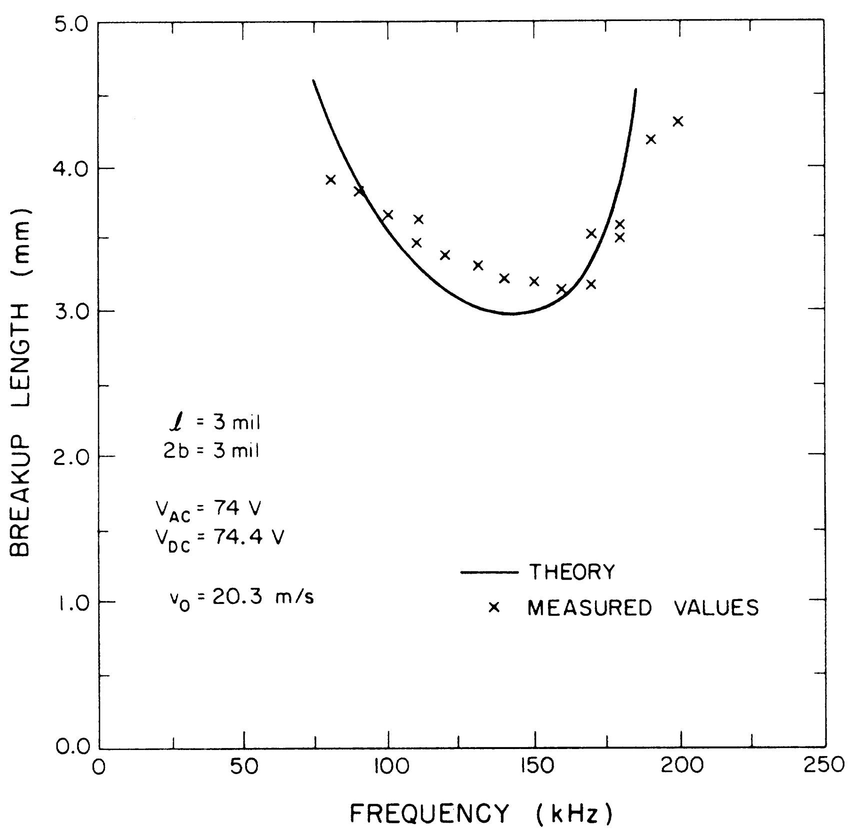

Fig. 4. Frequency response for a 3 mil thickness.

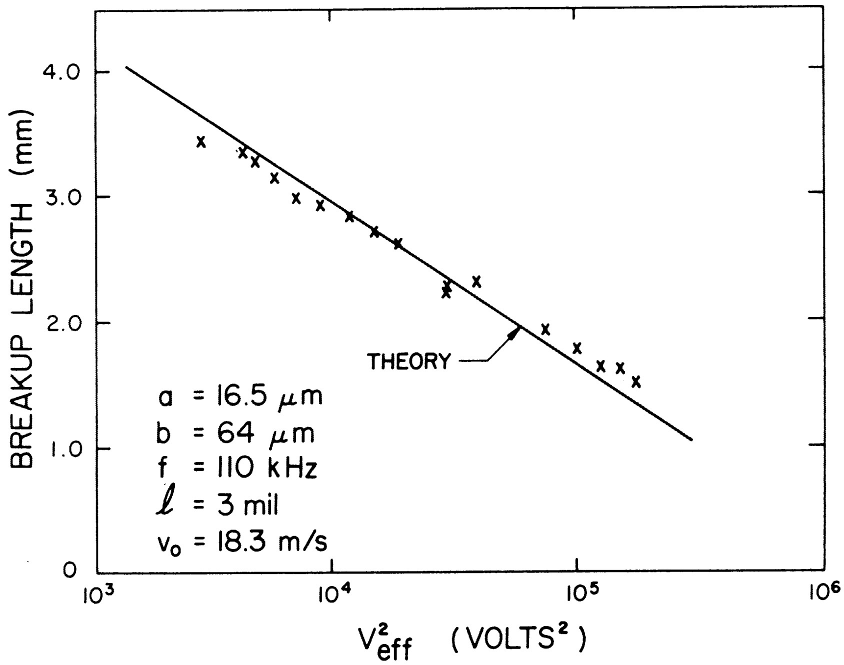

Fig. 5. Breakup length versus effective voltage.

mind that the theory, which rests on the fundamental equations of electrostatics and fluid mechanics, contains no adjustable parameters. The breakup length is predicted only in terms of geometrical measurements, applied voltage, and material properties.

Compared to acoustic excitation, EHD excitation is often relatively weak, so that the magnitude of the drive is extremely important in practice. The theory presented above predicts that the excitation pressure is proportional to the square of the effective voltage, which suggests that a relatively large increase in the excitation may be obtained by increasing the voltage at which the exciter operates. This prediction was tested experimentally by varying both the AC and DC voltage levels over a wide range, and measuring the breakup length.

Since the breakup length depends logarithmically on the excitation, a plot of breakup length against the logarithm of the effective voltage should give a straight line. Experimental values of breakup length are plotted in this way in Fig. 5, along with the predictions of the theoretical model. Both the magnitude and slope of the breakup length follow the predictions well, although the slope of the line appears to be somewhat flatter than expected. These results give us some confidence in extrapolating the design to even higher voltages to achieve a shorter breakup length if necessary, although these lengths are already comparable to those used in acoustic excitation.

6. Discussion

In EHD droplet generators, the electric force acts directly on the surface of the jet, and the nozzle assembly is not involved at all. The electrostatic interaction is quite straightforward, and can be described by standard field solutions, leading to a model of the excitation based on the fundamental equations of fluid mechanics and electrostatics. Even the approximate models developed in this report allow the complete design of exciters giving breakup lengths within a few percent of the desired value, an accuracy which is impossible with acoustic excitation.

A further advantage of EHD excitation lies in the relatively smooth frequency response. Unlike acoustic excitation, changes in the frequency over a range of several percent have virtually no effect on the breakup length of the jet, so careful tuning of the driver may be dispensed with.

EHD excitation if often thought to be too weak for practical droplet generators. When large electrodes are brought to the surface of the jet, breakdown occurs before a reasonably short breakoff length can be reached. The improvements in EHD excitation reported here came from the discovery that the basic exciter had to be much smaller than used before, and positioned away from the nozzle. In addition to furnishing a more efficient excitation, this arrangement allowed the use of electric fields much higher than those commonly thought to give air breakdown.

In the experiments, the shortest breakup lengths were limited by the power supplies, and not by breakdown, although the electric field at the surface of the jet was calculated to exceed 15 MV/m, far above the generally accepted breakdown value of 3 MV/m for large parallel electrodes. Higher breakdown fields are not uncommon with very small gaps, and have been noted for some time. The effect (often called the Paschen effect) is extremely useful in the EHD exciter, since it combines with the short optimal length to produce a compact, powerful exciter. In previous attempts to produce an EHD droplet generator, the electrodes were relatively large (on the order of centimeters), and prone to break down at low voltages. At the same time, these large electrodes would not have been more effective than the half wavelength electrode used in the present work, so their best output was disappointingly low. With shorter electrodes, and the higher electric pressures this allows, EHD excitation can produce output comparable to acoustic excitation.

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Notation

A	a constant of integration
a	unperturbed jet radius, m
b	exciter electrode radius, m
E	electric field at jet surface, V m^{-1}
i	$= \sqrt{-1}$
k	wavenumber, m^{-1}
l	exciter length, m
l_b	jet breakup length, m
p	pressure, N m^{-2}
s	Laplace transform variable, m^{-1}
R	jet radius, m
r	radial coordinate, m
T	surface tension, N m^{-1}
t	time, s
U	velocity, m s^{-1}
U_0	equilibrium jet velocity, m s^{-1}
u	$= (U/U_0) - 1$
V	electric potential, V
V_{eff}	effective value of voltage at a particular frequency, V
x	position along jet, m
α	relative effect of surface tension on perturbation (eqn. 10)
Δ	impulse function
δ	$= R - a$, displacement of jet, m
δ_0	displacement at exciter exit, m
δ'	convective radial velocity (eqn. 17), m s^{-1}
δ'_0	velocity at exciter exit, m s^{-1}
ϵ	electric permittivity, F m^{-1}
λ	wavelength, m
π	= 3.14159 . . .
ρ	density, kg m^{-3}
ω	frequency, rad s^{-1}

Marks

$\hat{}$ denotes complex amplitude

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