Role of Joule heating in the electrostatic spraying of liquids

Joseph M. Crowley

Department of Electrical Engineering, University of Illinois, Urbana, Illinois 61801

(Received 6 February 1976)

Electrostatic spraying from a liquid cone is associated with a high electric current density at the tip of the cone. For all nonmetals, and for some metals, the Joule heating produced by this current is sufficient to boil the liquid. It is suggested that boiling, rather than electrostatic force, is responsible for the liquid droplets which appear in electrostatic spraying. This suggestion is confirmed by comparison of the expected temperature with the observed mode of spraying for various metals and nonmetals.

PACS numbers: 47.55.Kf, 41.80.Gg, 47.25.Qv

INTRODUCTION

Electrostatic spraying of liquids has been observed since Zeleny,[1] but it remained a laboratory curiosity until recently, when it found importance in such diverse fields as rocket propulsion,[2] spray painting[3] and coating,[4] and high-voltage transmission lines.[5] Most of the liquids reported in the literature produce a spray consisting of charged droplets and ions, the relative proportions varying with operating conditions. Recently, however, several laboratories have reported the electrostatic production of ions from liquid cones without the accompanying charged droplets.[6,7] These ion sources use various liquid metals and alloys, such as sodium, NaK, and GaIn.

In all of these ion sources, the liquid metal forms a cone of the general shape predicted by Taylor[8] in which electrostatic forces are just balanced by the surface tension of the liquid. The ion spray emanates from the tip of the cone, where the field is highest, any may reach a current level of several hundred microamperes Since the tip of this cone is thought to be on the order of an atomic radius, there is a good possibility that this relatively large current flowing through such a small region might cause a significant temperature rise at the tip by Joule heating.

TEMPERATURE IN A SPRAYING CONE

To test this idea, the heat-flow equation was solved in the geometry of Fig. 1, which is a simplified model of the tip of the cone truncated at $r=a$, a distance which is presumed to be at least equal to the atomic radius of the liquid. The current i which flows as a result of field emission or some other discharge process from the tip of the cone is assumed to be distributed uniformly across the liquid cone of conductivity σ. The current density is therefore

$$J_r = -\frac{i}{2\pi r^2(1-\cos\alpha)} \tag{1}$$

independent of any charge buildup. This current density increases rapidly as the tip of the cone ($r=a$) is approached. The electric field in the liquid is

$$E_r = \frac{J_r}{\sigma} = -\frac{i}{2\pi\sigma r^2(1-\cos\alpha)} \tag{2}$$

by Ohm's law.

The current flow leads to heating of the liquid, since it represents a distributed heat source of strength

$$p = J\cdot E = \frac{i}{4\pi^2\sigma r^4(1-\cos\alpha)^2}. \tag{3}$$

The resulting temperature of the liquid in the cone can be determined by the heat-flow equation

$$\nabla^2 T = -\frac{i^2}{4\pi^2\sigma k(1-\cos\alpha)^2}\frac{1}{r^4}, \tag{4}$$

where the convection of heat by fluid motion is neglected. The constant k is the thermal conductivity of the liquid. The solution, assuming conical symmetry and neglecting heat transfer normal to the cone's surface, is

$$T = -\frac{i^2}{8\pi^2\sigma k(1-\cos\alpha)^2 r^2} + \frac{A}{r} + B. \tag{5}$$

The constants A and B are determined by the boundary conditions. Far from the tip ($r\to\infty$), the liquid approaches room temperature, so $B=T_0$. The heat flow away from the tip is $F=-k\nabla T$ or

$$F = \frac{kA}{r^2} - \frac{i^2}{4\pi^2\sigma(1-\cos\alpha)^2 r^3}. \tag{6}$$

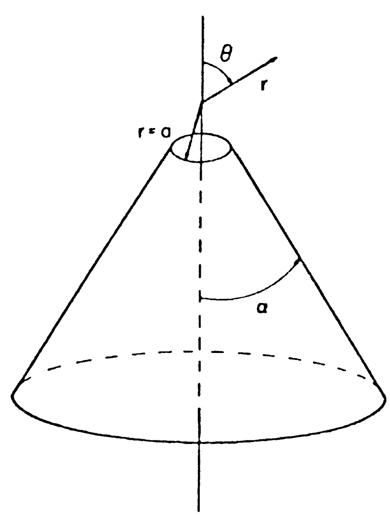

FIG. 1. A liquid cone of half-angle α, with an electric discharge at the blunted tip ($r=a$).

TABLE I. Some properties of liquids used in spraying.

	Liquid	Thermal cond. k (W/cm °C)	Elec. resistivity ρ (Ω cm)	Atomic radius r_A (Å)	Pure ion spray ever observed
Nonmetals	Doped glycerine	0.00267 [a]	10^3-10^5 [b]	...	No
	Salt water	0.00599 [a]	$1-10^5$ [a]	...	No
Metals	Hg	0.075 [a]	98.5×10^{-6} [a]	1.55 [c]	No
	Wood's metal	0.01335 [a]	52×10^{-6} [a]	1.1–1.75 [c]	No
	NaK	0.25 [d]	40×10^{-6} [d]	2.31–1.86 [c]	Yes [c]
	Cs	0.1841 [e]	36.6×10^{-6} [a]	2.62 [c]	Yes [c]
	Na	0.8 [d]	10.2×10^{-6} [a]	1.86 [c]	Yes [c]

[a] Reference 9.
[b] References 10 and 11.
[c] Reference 7.
[d] Reference 12.
[e] Reference 13.

Since the cone is truncated at $r=a$, we will take the condition that the heat flow at this point vanishes. While this is certainly not true, the exact nature of the boundary condition is of little interest, since the expected correction would not be significant in this rough model. Setting the heat flow at the tip equal to zero gives

$$\left.\frac{dT}{dr}\right|_{r\to a}=\frac{kA}{a^2}-\frac{i^2}{4\pi^2\sigma(1-\cos\alpha)^2a^3}=0, \tag{7}$$

so that the temperature distribution inside the liquid may be written

$$\Delta T=T-T_0=\frac{i^2}{4\pi^2\sigma(1-\cos\alpha)^2k}\left(\frac{1}{ar}-\frac{1}{2r^2}\right). \tag{8}$$

The maximum temperature of the liquid, which occurs at the tip, is given by

$$\Delta T_{\max}=\frac{i^2}{8\pi^2\sigma(1-\cos\alpha)^2a^2k}. \tag{9}$$

Using the value given by Taylor[8] for the angle of the cone, $\theta=49.3°$, the temperature rise at the tip may be expressed as

$$\Delta T_{\max}=0.105\left(\frac{\rho}{k}\right)\frac{i^2}{a^2}, \tag{10}$$

where ρ is the Ω cm, k is in W/cm sec, i is in A, and a is in cm.

DISCUSSION

The temperature rise depends on two related transport properties of the liquid, the electrical resistivity and the thermal conductivity. The fluids most used for electrostatic spraying may be divided into two groups with respect to these properties (see Table I). The first group is composed of nonmetals such as water and glycerine, which have low thermal conductivities and varying but high resistivities, depending on the impurities present. The right-hand side of Fig. 2 shows the temperature rise for a discharge current of 1 μA as a function of the tip's location for several nonmetals with typical resistivities.

For distances on the order of microns, the temperature becomes very high; higher in fact than the boiling point for these liquids at atmospheric pressure. It seems clear that such a condition cannot persist very long without serious changes in the nature of the spraying cone. Such changes will come in liquid properties such as surface tension and viscosity or eventually in the physical state of the cone, which will soon begin to boil. If this boiling is vigorous, large pieces of the cone will be thrown off, providing the charged droplets which appear in the spray. The loss of large pieces of liquid will cool the cone, and also stop the electric discharge, which depends on the high curvature of the tip. As the cone reforms, however, the electric discharge restarts, heating the cone, and leading to a repetition of the cycle. This suggested sequence for spray formation offers an alternative to the commonly accepted hypothesis of electrohydrodynamic instability of the liquid surface, in which pieces of the charged liquid are torn off by electric forces.

If the Joule heating of liquid metals is considered, however, the predicted temperatures are orders of magnitude lower, as shown in the left-hand side of Fig. 2. Here a 1-μA current leads to temperature rises which range from 0.15 to 1.5 °C at a distance of 1 Å from the tip. Since this distance is on the order of an atomic radius, it is not likely that the liquid properties will change significantly, or that boiling will occur. This suggests that the spray produced under these conditions will consist solely of the ions and electrons produced by the discharge process, without any charged liquid droplets.

This suggestion is borne out experimentally since only liquid metals have produced pure ion beams. Of course not all liquid metals produce this type of spray,

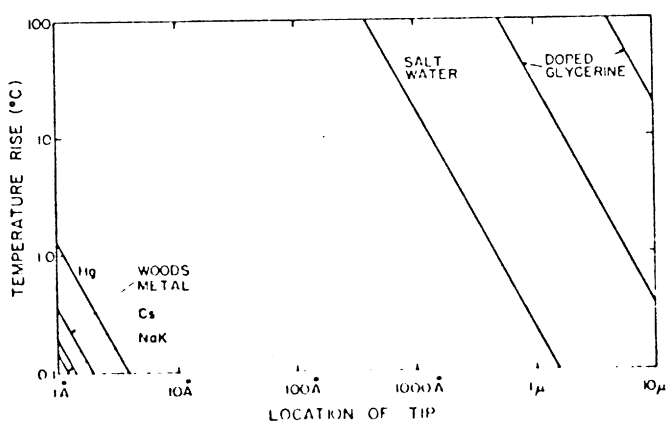

FIG. 2. The maximum temperature rise produced by 1 μA flowing through a truncated cone for various liquids as a function of the distance from the tip of the cone to the origin.

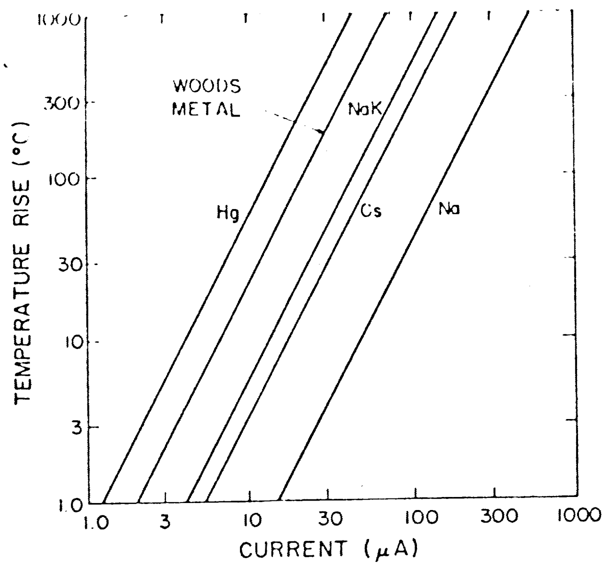

FIG. 3. The maximum temperature rise in a cone truncated at a distance of one atomic radius from the tip in various liquid metals, as a function of the discharge current.

but this may be expected because different metals have different thermal conductivities and electrical resistivities, so that the temperature at the tip of the cone may well be different for different metals. In addition, the current discharge from the tip is a strong function of the applied voltage as well as the metal properties and the geometry of the cone.

In order to place the various metals on a common footing, the predicted temperature rise for a cone truncated at a distance of one atomic radius from the tip [Eq. (10)] was plotted as a function of the discharge current in Fig. 3. The current which causes a sizable temperature rise (~100 °C) varies from 13.2 μA for mercury to 161 μA for sodium. Stated differently, a 20-μA current would cause a temperature rise in mercury of 228 °C, while the same current flow through sodium would only cause a 1.5 °C rise. The mercury tip would show strong thermal effects, and perhaps boiling in a vacuum, while the sodium tip would barely be affected. In practice, it is found that sodium can produce an ion beam free of droplets, while mercury always produces droplets.

Figure 3 also suggests that the metals may be ranked in order of temperature at the tip for a given current as Hg > Wood's metal > NaK > Cs > Na. Mahoney et al. have reported that purely ionic discharges have not been produced with mercury or Wood's metal, but are possible with NaK, cesium, and sodium, which is consistant with this ranking.

SUMMARY

Charged droplet sprays may be expected if Joule heating of a liquid in a Taylor cone raises the temperature sufficiently to alter the physical properties of the liquid, or to induce boiling. Such large temperature rises will always occur in liquid nonmetals, even with relatively small currents.

With liquid metals, however, the temperature rise for a cone truncated at one atomic radius from the tip may be negligibly small, depending on the metal and the current flow. If so, a spray composed solely of ions or electrons may be expected.

Experimentally, purely ionic beams have never been produced in nonmetals, as expected from calculation of the temperature rise at the tip. In addition, metals which show large temperature rises, such as mercury and Wood's metal, have never shown pure ionic beams, while those with small rises, such as NaK, sodium, and cesium, have all produced ionic beams.

[1]J. Zeleny, Proc. Cambridge Philos. Soc. 18, 71 (1915).
[2]Prog. Astronaut. Rocketry v. 5 (1961).
[3]E. P. Miller in *Electrostatics and Its Applications*, edited by A. D. Moore (Wiley, New York, 1973), p. 250.
[4]D. Michelson, J. Fluid Mech. 33, 573 (1968).
[5]J. F. Hoburg and J. R. Melcher, IEEE Trans. Power Appar. Syst. PAS-94, 128 (1975).
[6]D. S. Swatik and C. D. Hendricks, AIAA J. 6, 1596 (1968).
[7]J. F. Mahoney, A. Y. Yahiku, H. L. Daley, R. D. Moore, and J. Perel, J. Appl. Phys. 40, 5101 (1969).
[8]G. I. Taylor, Proc. R. Soc. (London) A 280, 383 (1964).
[9]*Handbook of Chemistry and Physics* (The Chemical Rubber Co., Cleveland, 1936).
[10]P. W. Kidd, J. Spacecr. Rockets 5, 1034 (1968).
[11]M. W. Huberman, J. Spacecr. Rockets 5, 1319 (1968).
[12]*Liquid Metal Handbook*, edited by C. B. Jackson (U. S. GPO, Washington, D. C., 1955).
[13]*The Encyclopedia of the Chemical Elements*, edited by C. A. Hampel (Reinhold, New York, 1968).
[14]R. J. Pfeifer and C. D. Hendricks, Phys. Fluids 10, 2149 (1967).
[15]H. Miyahara, J. Phys. Soc. Jpn. 27, 1062 (1969).
[16]V. E. Krohn, J. Appl. Phys. 45, 1144 (1974).